CONSTRAINTS ON THE INTERGALACTIC TRANSPORT OF COSMIC RAYS

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ABSTRACT

Motivated by recent experimental proposals to search for extragalactic cosmic rays (including anti-matter from distant galaxies), we study particle propagation through the intergalactic medium (IGM). We first use estimates of the magnetic field strength between galaxies to constrain the mean free path for diffusion of particles through the IGM. We then develop a simple analytic model to describe the diffusion of cosmic rays. Given the current age of galaxies, our results indicate that, in reasonable models, a completely negligible number of particles can enter our Galaxy from distances greater than ~ 100 Mpc for relatively low energies ($E < 10^6$ GeV/n). We also find that particle destruction in galaxies along the diffusion path produces an exponential suppression of the possible flux of extragalactic cosmic rays. Finally, we use gamma ray constraints to argue that the distance to any hypothetical domains of anti-matter must be roughly comparable to the horizon scale.

Subject headings: ISM: Cosmic Rays – Elementary Particles – Cosmology: Theory – Intergalactic Medium

1. Introduction

Cosmic rays of extragalactic origin can potentially provide an important probe of our universe. However, the propagation of cosmic rays is highly constrained by magnetic fields, both in the interstellar medium and in the intergalactic medium (IGM). In this paper, we outline the basic issues involved in cosmic ray propagation through the IGM and into the Galaxy. In particular, we show that the total distance traveled by cosmic rays during the age of the universe is severely limited. These results have implications for recently proposed experiments to detect anti-matter and ordinary cosmic rays from external galaxies (see also Ormes et al. 1997).

The framework for this paper can be summarized as follows. [1] In the intergalactic medium, we assume that cosmic rays diffuse with a characteristic mean free path. By varying the mean free path, one can investigate particle propagation in a large number of physical scenarios. [2] Particles have a limited accessibility to individual galaxies. This accessibility may depend upon galactic winds and/or magnetic barriers. [3] The propagation of cosmic rays depends on the particle energies. For low energy cosmic rays, $E \sim 1 - 10$ GeV/n, the particles are likely to follow magnetic field lines. For high energy particles, $E \gg 10^{18}$ eV/n, the particles no longer follow field lines and random walk through space. For intermediate energies, the particle propagation problem is much harder to describe. In any case, the dependence on cosmic ray energy should be kept in mind throughout this discussion.

We note that the global magnetic field structure of the universe remains uncertain. The goal of the paper is to explore the optimal case for transport of particles to our Galaxy from long distances. Even with the most optimistic plausible choice of parameters, we find that the particles cannot propagate farther than a few hundred Mpc during the lifetime of the universe. Our analysis using this framework will be useful for evaluating the feasibility of various detection strategies for future experiments.

This paper is organized as follows. We first estimate the magnetic field strength and discuss the corresponding constraints on the mean free path for cosmic rays (§2). In §3, we develop a simple analytic model to describe the self-similar diffusion of cosmic ray particles through the IGM; we use this model to estimate the abundances of extragalactic cosmic rays and anti-matter. We also use gamma ray constraints to estimate the distance to hypothetical domains of anti-matter. We conclude in §4 with a summary and discussion of our results.

2. Magnetic Field Limits and Mean Free Paths

In this section, we discuss the field strength and the coherence length for magnetic fields in the IGM. Magnetic fields greatly influence the motion of charged particles such as cosmic rays, provided that the particle pressure is small compared to the magnetic pressure (as we assume here). We can consider two different regimes of interest. For cosmic rays of sufficiently low energy, the magnetic gryo radius r_B is small compared to the coherence length of the magnetic field; in this limit, particles tend to follow the field lines and the magnetic field geometry determines the paths taken by the particles. In the opposite limit of high energy cosmic rays, the gyro radius is large compared to the coherence length; in this case, the particles exhibit a random walk with a mean free path comparable to the gyro radius, i.e., $\ell \sim r_B$. The magnetic field strength thus determines the energy boundary between low energy and high energy particles. In addition, for high energy particles, the field strength determines, in part, the mean free path.

We thus need an estimate of the magnetic field strength in the IGM. We first consider the simplest theoretical considerations. Given that the galactic field strength is $B_{\rm gal} \approx 1 - 3\mu$G (Heiles 1976), we can estimate the magnetic field strength B_{IGM} between galaxies in the following two ways:

[1] We consider the flux freezing approximation, in which the galaxy formed from a much larger region with a magnetic flux Φ. Using standard flux freezing arguments (zero resistance, infinite conductivity, the magnetic flux $\Phi = BR^2 = \textit{constant}$), we obtain

$$B_{IGM} = B_{\rm gal}\Big(\frac{\rho_{\rm gal}}{\rho_{IG}}\Big)^{-2/3}(1+z)^{-2} \sim 10^{-9}\,{\rm G}\,, \tag{1}$$

where $\rho_{\rm gal}$ is the typical density of the galaxy and ρ_{IG} is the density of the IGM. The factor $(1+z)^2 \approx 16$ takes into account the expansion of the universe since galaxies formed.

[2] Far from the galaxy, the leading order term in the multipole expansion for the magnetic field strength is the dipole term, which decreases with radius as r^{-3}. Between galaxies, the magnetic dipole term thus contributes a characteristic field strength

$$B_{IGM} = B_{\rm gal}\Big(\frac{\rho_{IG}}{\rho_{\rm gal}}\Big) \sim 10^{-12}\,{\rm G}\,. \tag{2}$$

If B_{IGM} is much smaller than this fiducial value, the galactic field geometry must have a rather special form so that the dipole term vanishes (or is highly suppressed).

We note that other effects can increase the magnetic field strengths between galaxies beyond the simple estimates given here. For example, galactic scale winds can drag magnetic field lines into the intergalactic medium and thereby enhance the field strength of the IGM (Kronberg & Lesch 1997). Many other magnetohydrodynamical effects can also take place, including field diffusion, reconnection, and dynamo activity (see, e.g., Shu 1992).

Observations of magnetic fields are roughly consistent with the estimates given above. Although the magnetic field in our galaxy is relatively well observed (e.g., Heiles 1976), magnetic fields are just now being studied in external galaxies and the IGM (see, e.g., the reviews of Kronberg 1994 and Biermann 1997). Measurements of magnetic fields in other galaxies suggest that our galaxy is fairly typical and that field strengths of $B \approx 3 - 10$ μG are the norm. Within galaxy clusters, magnetic fields can now be measured (e.g., Kim et al. 1990) and the typical field strength in the central regions is $B \sim 2$ μG. Furthermore,

the characteristic length for field reversal is 13–40 kpc or perhaps even smaller (see Feretti et al. 1995). If we scale these values to larger size scales using a flux freezing argument, the estimated IGM field strength is comparable to that of equation [1]. Unfortunately, however, very little data exist to constrain magnetic fields in the less dense regions of the universe, i.e., outside the core regions of clusters (see Ensslin et al. 1997 and Biermann 1997). In one case, outside the Coma cluster, the magnetic field strength has been estimated to be as large as 10^{-7} G (Kim et al. 1989). In addition, across cosmological distances, there is an upper limit on the magnetic field strength, $B_{IGM} < 10^{-9}$ G, which is estimated from rotation measure data along the line of sight to quasars and modeling (Kronberg 1994; see also Vallée 1990).

Nearly independent of their origin, cosmic magnetic fields will be pulled along by baryonic matter and will thus be tied to galaxies. Using this idea, models of the intergalactic magnetic fields are now being proposed (see, e.g., Biermann, Kang, & Ryu 1996; Kronberg 1996; Kronberg & Lesch 1997). This work shows that magnetic fields lines can in principle connect galaxies to each other and that the magnetic field strength can lie in the range 10^{-7} – 10^{-10} G, consistent with the simple estimates given above. Notice also that the field lines between galaxies will tend to straighten out due to both magnetic tension and the expansion of the universe.

To summarize, we take the magnetic field strength in the IGM to lie in the range 10^{-12} G $< B_{IGM} < 10^{-7}$ G. The lower limit arises because of the dipole contribution from galaxies. The upper limit is indicated both by observations and by elementary physical considerations.

We now estimate mean free paths for cosmic rays in the IGM. For most of this paper, we use the limiting case in which the mean free path ℓ is the typical distance between galaxies, i.e., $\ell \approx 1$ Mpc. This choice provides an optimistic but plausible case for the transport of low energy cosmic rays. This limit is realized, for example, if the magnetic field lines are straight between galaxies and no other effects impede the propagation of particles. Then

the intergalactic field lines are tied to the galactic field lines, so the coherence length of the field can be as large as the mean separation between galaxies. However, galaxies are randomly oriented and it is thus highly unlikely that the magnetic field reversal length scale is much larger than 1 Mpc. Notice that the exact value of the magnetic field strength is not important in this case.

We also consider the opposite limit in which the magnetic field is extremely tangled and the cosmic rays are well coupled to the field. In this case, the effective mean free path ℓ is given by the magnetic gyro radius r_B, i.e.,

$$\ell = \lambda r_B = \lambda \frac{E}{qB} = 1\,\mathrm{pc}\,\Big(\frac{E}{1\mathrm{GeV}}\Big)\Big(\frac{B}{10^{-12}\mathrm{G}}\Big)^{-1} \lambda\,, \tag{3}$$

where E is the energy of the particle, q is the charge, and B is the magnetic field strength. The dimensionless enhancement factor λ takes into account the possibility that the mean free path can be somewhat longer than the magnetic gyro radius.

For typical cosmic ray energies, $E \sim 1$ GeV/n, where n is the number of nucleons, the magnetic gyro radius is a factor of $\sim 10^6$ smaller than the most probable distance between galaxies. Thus, to estimate the maximum possible influence of extragalactic cosmic rays, we use the distance between galaxies $\ell \sim 1$ Mpc as the optimal mean free path. However, cosmic rays with energies larger than about $E \sim 10^6$ $(B/10^{-12}\mathrm{G})$ GeV have magnetic gyro radii larger than 1 Mpc; these high energy cosmic rays experience a completely tangled magnetic field and thus have a mean free path given by equation [3].

Given the mean free path for both high and low energy cosmic rays, we can now estimate how far these particles travel in a given time period. For diffusion in a uniform medium (e.g., the IGM), the distance traveled in a time interval t is roughly given by the diffusion length R_0, which we write in the form

$$R_0 = [D\,t]^{1/2} = [c\,t\,\ell/3]^{1/2} = 32\,\mathrm{Mpc}\,(\ell/1\mathrm{Mpc})^{1/2}\,(t/10\mathrm{Gyr})^{1/2}\,, \tag{4}$$

where c is the particle speed (the speed of light), and ℓ is the mean free path. Thus, in the age of the universe, these particles have traveled a distance much smaller than the horizon size, $\sim 3000h^{-1}$ Mpc.

3. A Model for Cosmic Ray Diffusion

In order for cosmic rays from large distances to enter our Galaxy, they must overcome two hurdles: [1] The particles must first diffuse through the IGM to reach the general vicinity of our Galaxy. [2] The particles must enter the Galaxy itself by overcoming the barriers provided by magnetic fields, galactic winds, and other effects. In this section, we develop a model for cosmic ray diffusion and include the effects of the fractional accessibility to the galaxy.

This model is based on the simplified picture of intergalactic magnetic fields as discussed in §2. It is important to keep in mind that cosmic ray propagation depends on the particle energy. In this picture, low energy cosmic rays follow field lines from galaxy to galaxy with a mean free path of ~ 1 Mpc; most of this section deals with this low energy case. On the other hand, high energy cosmic rays perform a random walk that is almost independent of the galaxies; we consider the high energy limit in §3.6. We also note that other magnetic field configurations are possible. However, this present formulation is rather robust in that it can be applied to many different specific models for magnetic field configurations and cosmic ray propagation, provided only that the evolution is diffusive.

3.1. A Self-Similar Diffusion Model

The cosmic ray flux from any given galaxy has a limited sphere of influence because of the relative difficulty for particles to diffuse through the IGM. The number density and the

flux of cosmic rays can be described by a simple similarity solution. We begin by writing the diffusion equation in the form

$$\frac{\partial n}{\partial t} = D\nabla^2 n - \Lambda n \, , \tag{5}$$

where $D = c\ell/3$ is the diffusion constant and the parameter Λ accounts for the destruction of cosmic rays. The solution for the number density of cosmic rays can be written in the form

$$n(r,t) = t^{-1/2} \, \mathrm{e}^{-\Lambda t} \, f(\xi) \, , \tag{6}$$

where the power-law index 1/2 is chosen so that the galaxy has a constant luminosity L_{CR} of cosmic rays. The similarity variable ξ is defined by $\xi \equiv r/R_0$, where R_0 is a characteristic length scale $R_0 \equiv [ct\ell/3]^{1/2} \approx 32$ Mpc $(\ell/1\mathrm{Mpc})^{1/2} \, (t/10\mathrm{Gyr})^{1/2}$ (see equation [4]).

Notice that we neglect the expansion of the universe in this simple treatment of the problem. This treatment is justified because the effective diffusion length (see below) is much smaller than the horizon size. In addition, the inclusion of the expansion of the universe introduces additional assumptions (e.g., an open versus closed universe).

With the above definitions, the resulting ordinary differential equation for the reduced diffusion field $f(\xi)$ takes the simple form

$$f_{\xi\xi} + \frac{2}{\xi} f_\xi + \frac{\xi}{2} f_\xi + \frac{1}{2} f = 0 \, , \tag{7}$$

where subscripts denote differentiation. The relevant boundary conditions for this problem are [1] the cosmic ray luminosity approaches a constant at the origin ($f \to 1/\xi$ as $\xi \to 0$), and [2] the cosmic ray flux outward through any given spherical shell vanishes at spatial infinity ($\xi^2 f \to 0$ as $\xi \to \infty$). With these boundary conditions, equation [7] has the solution

$$f(\xi) = f_0 \frac{1 - \mathrm{erf}(\xi/2)}{\xi} \, , \tag{8}$$

where the constant f_0 is determined by the cosmic ray luminosity of the galaxy and erf(z) is the error function. The flux $\mathcal{F}$ of cosmic rays at a distance r from the galaxy is given by $\mathcal{F} = -D\partial n/\partial r$. The cosmic ray luminosity L_{CR} of the galaxy is

$$L_{CR} = \lim_{\xi \to 0} 4\pi r^2 \mathcal{F} = \lim_{\xi \to 0} -4\pi D r^2 \frac{\partial n}{\partial r} = 4\pi f_0 (\ell c/3)^{3/2} \, . \tag{9}$$

It is useful to combine these results to write the cosmic ray flux in the form

$$\mathcal{F} = \frac{L_{CR}}{4\pi r^2} \, g(\xi) \, \mathrm{e}^{-\Lambda t} \qquad \text{where} \qquad g(\xi) \equiv \Big\{ 1 - \mathrm{erf}(\xi/2) + \xi \pi^{-1/2} \exp[-\xi^2/4] \Big\} \, . \tag{10}$$

The function $g(\xi)$ thus encapsulates the departure of the cosmic ray flux from the naive result $\mathcal{F} = L_{CR}/4\pi r^2$ which applies in the limit of no diffusion ($\ell \to \infty$). Note that this diffusive argument does not include energy loss of the particles from adiabatic cooling.

3.2. Fractional Accessibility Argument

We must take into account the possible destruction of cosmic rays as they travel through the universe. We have proposed a model in which cosmic rays propagate from galaxy to galaxy and finally reach our galaxy. Each galaxy along the way has some accessibility fraction x, i.e., the fraction of incident cosmic rays that actually enter the galaxy. Once inside a galaxy, cosmic rays bounce around for a time τ_{esc}, the escape time. The cosmic rays have some chance of interacting and being destroyed with a characteristic time scale τ_{int}. As a result, the fraction f_1 of cosmic rays that remain after each galactic visit (each step of the random walk) is given by

$$f_1 = (1-x) + \frac{x}{1 + \tau_{esc}/\tau_{int}} \equiv 1 - \alpha x \qquad \text{where} \qquad \alpha \equiv \frac{\tau_{esc}}{\tau_{int} + \tau_{esc}} \, . \tag{11}$$

We note that the destruction parameter α is energy dependent. For relatively low energies ($E < 10$ GeV/n), the interaction time and the escape time are comparable, $\tau_{int} \sim$

$\tau_{esc} \sim 10^7$ yr (Ormes & Freier 1978), and the parameter $\alpha \approx 1/2$. These results are based on solid experimental measurements. The escape time τ_{esc} is based on the abundance of the radioactive species ^{10}B and the interaction time τ_{int} is based on the well measured secondary to primary ratio (B/C). At very high energies, τ_{esc} and τ_{int} are not well determined experimentally. For energies $E > 10$ TeV/n, element separation becomes difficult and the secondary to primary ratio (B/C) becomes uncertain. However, one would expect that the escape time becomes comparable to the light crossing time of the galaxy, $\tau_{esc} \sim 3 \times 10^4$ yr, and α approaches a smaller value, at least 1/300.

As an aside, we note that an escape time $\tau_{esc} \sim 10^7$ yr, combined with the coherence length of the galactic magnetic field $\ell \sim 300$ pc, implies an effective diffusion length of $L_D = [c\tau_{esc}\ell]^{1/2} \approx 30$ kpc, a length scale comparable to the size of the galaxy. It is thus plausible that a diffusion model applies to the escape of cosmic rays from the galaxy. However, since we know the escape time τ_{esc} from experimental considerations, we need not use such a model for this paper.

On average, each cosmic ray will interact with $N \approx c\langle t\rangle/\ell$ galaxies, where the average time $\langle t\rangle$ since the cosmic ray was emitted is half the age of the galaxy and hence $N \approx 1500$ $(\ell/1\text{Mpc})^{-1}$. After N interactions, the total remaining fraction f_N of cosmic rays is $f_N = f_1^N = (1-\alpha x)^N$. In the large N limit, this expression approaches the form $f_N \approx \exp[-\alpha N x]$ and we make the identification $\Lambda t = \alpha x N$ to specify the destruction rate Λ in terms of the other parameters of the problem, i.e., $\Lambda = \alpha x c/2\ell$.

This interaction loss from the cosmic ray flux sets up an accessibility problem. In order for cosmic rays to survive the diffusion process, the fractional accessibility x must be very small. If the fraction x is small, however, the particles have little chance of entering our own galaxy. These two competing effects imply that the flux of cosmic rays into our Galaxy is

proportional to the “destruction function” F_D defined by

$$F_D(x) = x(1 - \alpha x)^N \, . \tag{12}$$

Here, the factor x is the probability that a cosmic ray will enter our Galaxy and the factor $(1 - \alpha x)^N$ is the probability of surviving other galaxies en route. Hence, the function $F_D(x)$ represents the fraction of the total possible cosmic ray flux that can enter our Galaxy. The destruction function has a maximum value $F_{\rm max} = \alpha^{-1}(N+1)^{-1}[N/N+1]^N \approx ({\rm e}\alpha N)^{-1}$ at the critical value $x = [\alpha(1+N)]^{-1}$. For typical values ($\alpha = 1/2$; $\ell = 1$ Mpc), we find $F_{\rm max} \approx 5 \times 10^{-4}$ at $x \sim 10^{-3}$. We obtain essentially the same result in the continuum limit using the form

$$F_D(x) \equiv x{\rm e}^{-\Lambda t} = x{\rm e}^{-\alpha x N} \tag{13}$$

for the destruction function. Notice that the mean number N of galaxies in the path of a cosmic ray depends on the mean free path, i.e., $N \approx 1500(\ell/1{\rm Mpc})^{-1}$. For most of parameter space, the function F_D is quite small and is sharply peaked about its maximum value. In particular, $F_D \to 0$ for both limiting cases $x \to 0$ and $x \to 1$.

The fractional accessibility x can in principle be calculated. In order to enter a galaxy, cosmic rays must propagate through galactic winds and any other inhibiting factors. One such calculation (Ahlen et al. 1982) uses a planar diffusion model to represent the disk of the galaxy; these authors find that the accessibility parameter x is close to 1/10 at GeV energies and approaches unity at high energies. Subsequent work using a spherical diffusion model found comparable results for x. The galaxy is expected to behave in a manner intermediate between the planar and the spherical models. In the spherical diffusion case, however, some adiabatic losses occur; these losses are analogous to the case of the solar wind. For the case of spherical diffusion, adiabatic losses shift the cosmic ray spectrum in energy by perhaps several GeV. Because of this effect, the cosmic rays one observes at a given energy represent the IGM spectrum at slightly higher energies. Since the flux of cosmic rays decreases rapidly

with energy, the net effect of these losses is to reduce the total flux at a given energy. This effect is of course most important at low energies comparable to the energy shift (perhaps several GeV). In this paper, however, we are interested in finding the largest possible cosmic ray flux from other galaxies, so we do not include these losses.

The energy shift of several GeV can be roughly approximated. The energy scale at which modulation becomes important can be estimated by the condition $VR/D = 1$, where $V \sim 10$ km/s is the speed of the galactic wind, $R \sim 10$ kpc is the size of the galaxy, and $D = c\ell/3$ is the diffusion coefficient (see Ahlen et al. 1984 for further discussion of all of these points). Using this relation, and scaling to the case of the solar wind, we find an energy shift of about 10 GeV.

3.3. Extragalactic Cosmic Rays

We now estimate the total flux of cosmic rays that are emitted by external galaxies and absorbed by our galaxy. The total flux $\mathcal{F}_T$ of extragalactic cosmic rays that impinge upon our galaxy is given by the integral

$$\mathcal{F}_T = L_{CR}\, n_{\rm gal}\, R_0\, {\rm e}^{-\Lambda t} \int_0^\infty g(\xi) d\xi = 4\pi^{-1/2}\, L_{CR}\, n_{\rm gal}\, R_0\, {\rm e}^{-\Lambda t}\,, \tag{14}$$

where $n_{\rm gal}$ is the number density of galaxies. Since this integral only has support in the local portion of the universe, we need not consider the curvature of the universe in this evaluation. We can now estimate the fraction of cosmic rays within our galaxy that have an extragalactic origin. The rate of absorption of extragalactic cosmic rays by our galaxy is given by

$$L_X = 2\pi R_D^2 x \mathcal{F}_T\,, \tag{15}$$

where x is the fraction of the incident cosmic rays that enter the galaxy (we assume that the galaxy can be modeled as a disk with radius $R_D \sim 15$ kpc). Within our Galaxy, the fraction

of cosmic rays χ that have an extragalactic origin becomes

$$\chi \equiv \frac{L_X}{L_{CR}+L_X} \approx \frac{L_X}{L_{CR}} = 8\pi^{1/2}[R_D^2 R_0 n_{\rm gal}]\, x {\rm e}^{-\Lambda t} \approx 0.1(\ell/1{\rm Mpc})^{1/2}\, x{\rm e}^{-\Lambda t}\,. \tag{16}$$

The result thus depends on the "destruction function" F_D defined above. Since the destruction function has a maximum value of $F_D = ({\rm e}\alpha N)^{-1} \sim 5\times10^{-4}$, the maximum expected fractional abundance of extragalactic cosmic rays is similarly small, i.e., $\chi = L_X/L_{CR} < 5\times10^{-5}$ (for a mean free path $\ell = 1$ Mpc). For most of parameter space, the fraction of extragalactic cosmic rays is extremely small.

3.4. Detecting A Possible Anti-Matter Signal

Ever since antiprotons were discovered (Chamberlain et al. 1955), searches for extragalactic anti-matter have steadily improved. Antiprotons of secondary origin have recently been found (e.g., Yoshimura et al. 1995; Mitchell et al. 1996), but heavier anti-matter which could originate in a cosmic anti-matter domain (primary anti-matter) has not been detected. The experimental situation prior to 1980 is summarized in Ahlen et al. (1982). Subsequent limits on $\overline{\rm He}/{\rm He}$ using balloon experiments have reached the 8×10^{-6} level (Ormes et al. 1997) and may ultimately reach 10^{-7} with the development of long duration ballooning. A planned space experiment, the Alpha Matter Spectrometer (AMS), may conceivably reach the 10^{-8} level (see, e.g., Ahlen et al. 1994). It is thus worthwhile to consider the possible importance of obtaining even tighter bounds on extragalactic anti-matter. This discussion assumes low energy cosmic rays, the regime accessible by future experiments. Furthermore, this treatment attempts to be optimistic in the sense that we try to find the largest possible anti-matter signal within a plausible class of models.

To consider cosmic rays emitted by distant (hypothetical) anti-galaxies, we find the

fraction $\mathcal{R}(a)$ of cosmic rays originating at distances greater than some scale a,

$$\mathcal{R}(a) \equiv \frac{\int_{\xi_a}^{\infty} g(\xi)d\xi}{\int_0^{\infty} g(\xi)d\xi} = \exp[-\xi_a^2/4] - \frac{\pi^{1/2}}{4}\,\xi_a\,[1-\mathrm{erf}(\xi_a/2)] \approx \frac{1}{2}\exp[-\xi_a^2/4]\,, \tag{17}$$

where $\xi_a = \xi(a) = a/R_0$. The final equality evaluates this ratio in the asymptotic limit ($\xi_a \to \infty$). For large distances $a \gg R_0$, the ratio $\mathcal{R}$ decays like a gaussian and hence the volume of the universe that produces cosmic rays accessible to our galaxy has a radius of $\sim 2R_0 \approx 64$ Mpc. The fractional abundance $\mathcal{A}$ of extragalactic anti-matter is given by the product $\mathcal{A} = \chi \mathcal{R} f_A$, where f_A is the fraction of anti-galaxies (for a baryon symmetric universe, $f_A = 1/2$). The expected fraction $\mathcal{A}$ of anti-matter in the cosmic ray flux can be written

$$\mathcal{A} = 0.025\,\tilde{\ell}^{1/2} x \exp[-(750x + 244\tilde{a}^2)/\tilde{\ell}]\,, \tag{18}$$

where the scaled variable $\tilde{\ell}$ is the mean free path in units of Mpc and $\tilde{a}$ is the distance to the nearest anti-galaxy in units of 1000 Mpc. In Figure 1, we have plotted the expected fractional abundance $\mathcal{A}$ of extragalactic anti-matter in the cosmic ray flux as a function of distance a to the nearest anti-matter domain.

Current experiments place rather tight constraints on the distance scale a to the nearest anti-matter domain (from Steigman 1976 to Cohen 1996). The strongest limits arise from considering the expected gamma ray flux from matter/anti-matter annihilations. The absence of copious gamma rays from galaxy clusters implies that $a > 40$ Mpc (e.g., Peebles 1993). Recent calculations (Cohen 1996) indicate that hypothetical anti-galaxies must be no closer than $a \approx 1500$ Mpc (see also Dudarewicz & Wolfendale 1994); this lower limit implies that the ratio $\mathcal{R} \sim 10^{-239}$ for mean free path $\tilde{\ell} = 1$ and for the optimal value of x. An independent calculation using the Sunyaev-Zel'dovich effect rules out anti-matter domains to a distance scale of $a \sim 200$ Mpc (Cohen, private communication). Finally, additional constraints can arise from annihilation signatures in the cosmic background radiation (see Kinney, Kolb, & Turner 1997).

To illustrate the difficulty associated with anti-matter domains, we present the following order of magnitude argument. We consider domain regions with size scale $\lambda \approx a$ and luminosity L_D in gamma rays from matter/anti-matter annihilation in the overlap regions. The total observed gamma ray flux is $F_\gamma \approx L_D (ct_0)^4 \, \lambda^{-6}$, integrated from the nearest domain boundary (at distance λ) out to the redshift at which the domains enter the horizon. The luminosity $L_D = g\dot{N}_N$, where $g \sim 4$ is the number of photons per annihilation and the annihilation rate per domain $\dot{N}_N = \epsilon \lambda_{100}^3 \, 5 \times 10^{55}$ s^{-1}. The efficiency ϵ is the fraction of the domain that annihilates during the age of the universe and λ_{100} is the domain scale in units of 100 Mpc. By comparing this expected gamma ray flux to the observed background flux, we obtain the constraint $\lambda_{100}^3 \geq 10^{14}\epsilon$. To estimate the efficiency ϵ, we assume spherical domains for which only the outer layer (of thickness ℓ_T) is available for annihilation: $\epsilon = 3\ell_T/\lambda$. For cosmological structures, the thickness ℓ_T should be of order the scale of galaxy formation, $\ell_T \sim 1$ Mpc. For this case, we obtain $\epsilon \sim 0.03\lambda_{100}^{-1}$ and hence a lower bound $\lambda_{100} > 1320$ (larger than the horizon size and hence unphysical). To allow nearby anti-matter domains ($\lambda_{100} \sim 1$), the efficiency must be very small $\epsilon \sim 10^{-14}$, which implies an unreasonably small domain thickness $\ell_T \sim 10^{12}$ cm (only ~ 10 stellar radii).

To evaluate the anti-matter fraction $\mathcal{A}$ for a given distance scale a, we first consider the maximal case by optimizing the function $\mathcal{A}$ with respect to the fractional accessibility x, i.e., $\mathcal{A} = 10^{-5}\tilde{\ell}^{3/2} \exp[-244\tilde{a}^2/\tilde{\ell}]$. Thus, for the representative values $\tilde{\ell} = 1 = \tilde{a}$, the maximum allowed anti-matter fraction is only $\mathcal{A} \sim 10^{-111}$. If we use a more typical value of the fractional accessibility $x = 0.1$, the anti-matter abundance $\mathcal{A}$ will be much smaller. Notice that the anti-matter fraction $\mathcal{A}$ has exponential sensitivity to the fractional accessibility x and has gaussian sensitivity to the distance scale a of anti-matter domains; this extreme sensitivity to the input parameters cannot be overemphasized (see Fig. 1). Indeed, the situation is such that no realistically conceivable improvement in experimental sensitivity to anti-matter could significantly increase the distance to which putative anti-matter domains

can be detected.

3.5. The Large ℓ Limit

For completeness and comparison, we consider the limit in which the mean free path becomes very large, i.e., the limit of no diffusion. This case is not expected to be realized in practice because any magnetic fields in the universe cause particle propagation to deviate from straight paths. Nonetheless, it is instructive to consider this limiting case. In this case, cosmic rays can travel straight to our Galaxy and we must consider the curvature of the universe. In this limit, the total extragalactic cosmic ray flux becomes

$$\mathcal{F}_T = L_{CR} \int n_{\rm gal} dr = L_{CR} c t_0 n_0 [(1+z)^{3/2} - 1] \, , \tag{19}$$

where we assume a spatially flat universe and $z \approx 3$ is the redshift at which galaxies begin to emit cosmic rays. The ratio χ of extragalactic to galactic cosmic rays becomes

$$\chi = \frac{L_X}{L_{CR}} = 2\pi x R_D^2 c t_0 n_0 [(1+z)^{3/2} - 1] \approx 30 x. \tag{20}$$

Thus, unless the accessibility fraction x is very small, extragalactic cosmic rays must provide a significant fraction of the total flux in the no diffusion limit. Notice that for cosmic rays with energies less than a few GeV, adiabatic losses will be significant and hence the flux of extragalactic cosmic rays will be highly suppressed. For cosmic rays with energies much higher than a few GeV, adiabatic losses are small and the above estimate is valid.

3.6. The High Energy Limit

We also consider the limit of high energy cosmic rays with $E \gg 1$ GeV. In this limit, the destruction parameter α, the fractional accessibility x, and the mean free path ℓ obtain different values than in the opposite (low energy) limit. These high energy cosmic rays

are not tightly bound to the galaxy and can escape on the light crossing time scale. At high energies, the destruction parameter becomes small, $\alpha \geq 1/300$, and the fractional accessibility x approaches unity. In this limit, the fraction χ of extragalactic cosmic rays obtains the simple form

$$\chi = L_X/L_{CR} = 0.1\,\tilde{\ell}^{1/2}\mathrm{e}^{-5/\tilde{\ell}}\,. \tag{21}$$

In the intermediate high energy regime, 1 GeV $\ll E \ll 10^6$ GeV, the mean free path $\tilde{\ell} \approx 1$, and the fraction of extragalactic particles approaches the constant value $\chi = L_X/L_{CR} \approx 7 \times 10^{-4}$.

At larger energies, $E \gg 10^6$ GeV $(B/10^{-12}$ G), the mean free path increases (see equation [3]) and the fraction of extragalactic cosmic rays increases accordingly. However, for energies less than $\sim 10^{18}$ eV, the galactic magnetic field scrambles the directions of extragalactic cosmic rays. Thus, a window exists for observing anisotropy in the extragalactic cosmic ray signal, 10^{18}eV $< E < 3 \times 10^{19}$eV, where the upper limit (the GZK limit) arises from the interaction of cosmic rays with the cosmic microwave background (Greisen 1966; Zatsepin & Kuzmin 1966). In this window, for $B_{IGM} = 10^{-9}$ G, the fraction χ lies in the range $7 \times 10^{-4} < \chi < 0.46$ according to this model; these χ values provide upper limits on the anisotropy. Future experimental searches for extragalactic cosmic rays should thus concentrate on the high energy regime (for the current experimental situation regarding arrival directions of high energy cosmic rays, see, e.g., Stanev et al. 1995; Watson 1996).

The validity of a diffusion model requires that the cosmic rays experience a large number of scattering events on their way to our galaxy. In the high energy limit considered here, the number N of scatterings depends on the magnetic field strength, where $N = ct/2\ell$ and ℓ is now given by the magnetic gyro radius. For IGM fields with large but representative field strengths, $B_{IGM} = 10^{-9}$ G, the number $N \approx 50$ at the highest energies, the GZK limit. The diffusion approximation thus remains marginally satisfied. However, for a weaker field,

$B_{IGM} = 10^{-12}$ G, the number N of scatterings becomes of order unity for particle energies greater than $E \sim 10^{18}$ eV. In this case, for relatively weak intergalactic magnetic fields, the considerations of the previous subsection apply.

3.7. Long Term Evolution

We have shown that any galaxy will have a rather small sphere of influence from its cosmic ray output. In particular, this sphere of influence (given by $R_0 \sim 32$ Mpc $(\ell/1\text{Mpc})^{1/2}$ $(t/10\text{Gyr})^{1/2}$) is much smaller than the horizon size scale ($\sim 3000\text{h}^{-1}$ Mpc), but much larger than the characteristic distance between galaxies (~ 1 Mpc). In order to gain further understanding of this problem, we consider the future evolution of cosmic ray diffusion in the universe for time scales that exceed the current Hubble time (see, e.g., Adams & Laughlin 1997 for a recent review of long term effects in the universe). The co-moving diffusion length can be written in the form

$$\widetilde{R}_0 = \frac{R_0}{R(t)} = \Big[\frac{ct\ell_0}{3R(t)}\Big]^{1/2} , \tag{22}$$

where $R(t)$ is the scale factor of the universe.

For a spatially flat universe, $R(t) \sim t^{2/3}$, and hence the co-moving diffusion length is a slowly growing function of time. Thus, the cosmic ray flux from a given galaxy will gradually influence galaxies of increasing distances. However, the co-moving diffusion length grows much slower than the horizon. As a result, the sphere of influence of any given galaxy will correspond to a decreasing fraction of the total volume of the observable universe as time proceeds. If the universe is open, then the scale factor approaches the form $R(t) \sim t$ in the relatively "near" future. In this case, the co-moving diffusion length approaches a constant value asymptotically in time. For completeness, we note that additional dynamical evolution of the magnetic fields will also take place.

4. Summary

We have presented a simple analytic model for the diffusion of cosmic rays through intergalactic space. This model clearly elucidates the difficulty faced by particles propagating large distances through the IGM. The results of this model are summarized below:

[1] The magnetic field strength estimated for the IGM lies in the range $B_{IGM} = 10^{-12}$ – 10^{-7} G. Low energy cosmic rays ($E < 10^6$ GeV) tend to follow the magnetic field lines. As a reasonable model for long distance particle transport, we take the cosmic ray mean free path to be comparable to the mean separation of galaxies, $\ell \sim 1$ Mpc. We take high energy cosmic rays to have a mean free path given by the magnetic gyro radius.

[2] The cosmic ray output of a galaxy has a sphere of influence with radius $R_0 = [ct\ell/3]^{1/2} \approx 32$ Mpc $(\ell/1\mathrm{Mpc})^{1/2}$ $(t/10\ \mathrm{Gyr})^{1/2}$. The time scale for particles to travel large distances ($r \gg R_0 \sim 32$ Mpc) through the IGM is thus much longer than the current age of the universe.

[3] We have demonstrated an accessibility problem for low energy extragalactic cosmic rays. In this model, cosmic rays diffuse through many different galaxies on the way to our Galaxy. In each galaxy, cosmic rays have some chance of being destroyed. In order for cosmic rays to survive the diffusion process, the fractional accessibility x must be small; if the accessibility x is small, however, cosmic rays have little chance of entering our Galaxy. This compromise sets up a maximum survival fraction of $F_{\max} \sim 5 \times 10^{-4}$.

[4] The fractional abundance of low energy extragalactic cosmic rays is extremely small for this model (eq. [16]). The abundance of extragalactic cosmic rays is exponentially suppressed by the fractional accessibility effect described in item [3].

[5] Since hypothetical galaxies made of anti-matter must be fairly distant ($a > 1000$ Mpc $\gg R_0$), the abundance of anti-matter in the cosmic ray flux corresponds to the gaussian tail

of the distribution. As a result, the fractional abundance of anti-matter is expected to be small (eq. [18] and Fig. 1) even for the extreme case of a baryon symmetric universe. Although anti-matter domains in the universe remain an interesting possibility, it must be realized that cosmic rays do not provide an effective search method.

[6] If cosmic rays propagate freely rather than diffuse, the fractional abundance of extragalactic cosmic rays would be much higher, as large as $30x$, where x is the fractional accessibility. This case is essentially ruled out by existing experimental constraints, but more definitive data will be forthcoming.

[7] We have shown that a window exists for observing cosmic ray anisotropy at high energies in the range $10^{18}\mathrm{eV} < E < 3 \times 10^{19}$ eV (see section 3.6).

[8] This formulation is in some sense more robust than its derivation because it has been posed parametrically. In particular, it can be applied to many different specific models for cosmic ray propagation and magnetic field configurations, provided that the evolution is diffusive.

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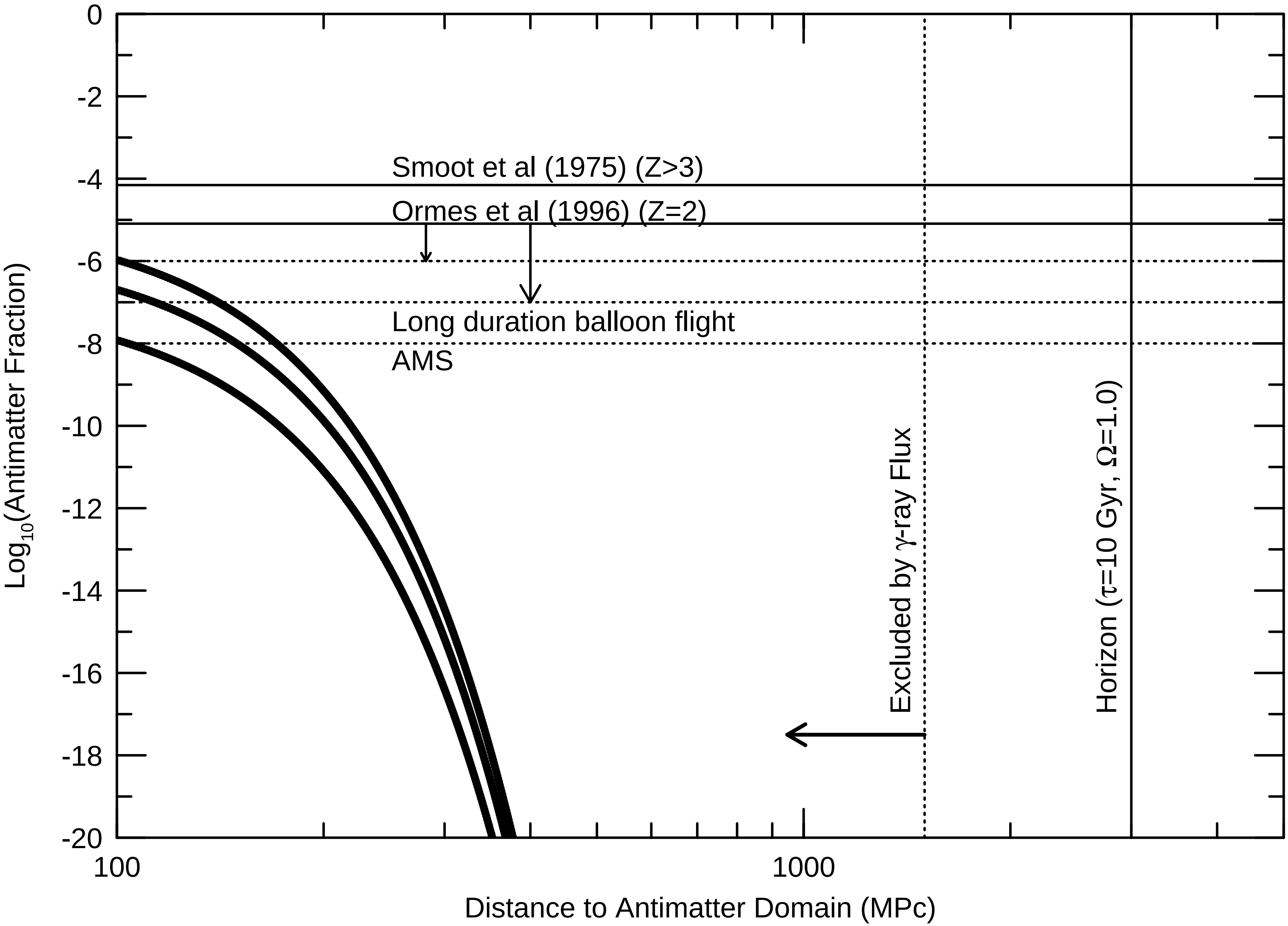
Figure Captions

Fig. 1.— Expected fractional abundance $\mathcal{A}$ of extragalactic anti-matter in the cosmic ray flux as a function of distance a to the nearest anti-matter domain. The curves at the left show $\mathcal{A}$ for mean free path $\ell = 1$ Mpc and varying values of the fractional accessibility x: top curve uses optimal value $x = 1.33 \times 10^{-3}$; middle curve uses $x = 10^{-4}$; bottom curve uses $x = 10^{-2}$. The curve using the most likely value $x = 0.1$ has abundance values $\mathcal{A}$ less than 10^{-20} over the entire distance range shown. The current best experimental limits are shown as solid horizontal lines, whereas the sensitivity of proposed measurements are indicated with horizontal dotted lines. The regions of parameter space excluded by γ-ray flux considerations (Cohen 1996) and the horizon distance are also indicated.



Smoot et al (1975) (Z>3)
Ormes et al (1996) (Z=2)
Long duration balloon flight
AMS
Excluded by γ-ray Flux
Horizon (τ=10 Gyr, Ω=1.0)
Log10(Antimatter Fraction)
Distance to Antimatter Domain (MPc)
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
100
1000